



Discrete
Math



Graph Theory

Hi

Lecture Objectives:

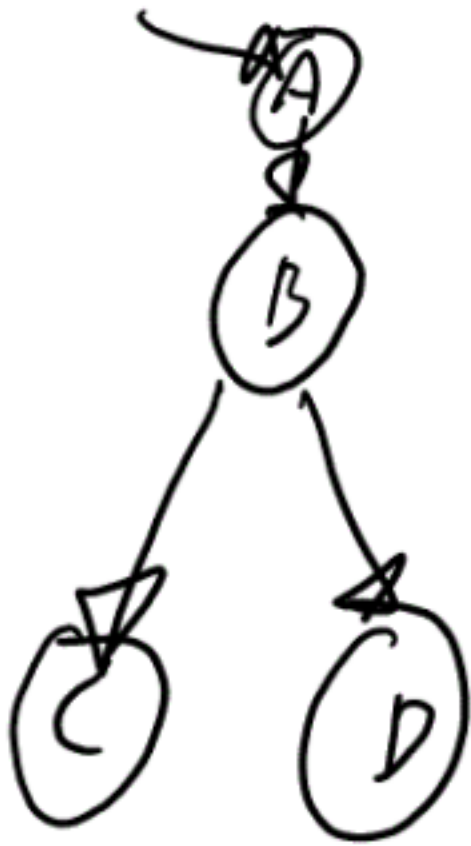
- 1) Define the term graph, node, initial node, final node, and edge.
- 2) Define reachability in a graph.
- 3) Define the term test path.
- 4) Explain the concept of a SESE graph.
- 5) Relate the concept of a SESE graph to good programming concepts.
- 6) Define the concept of visiting a node and explain the concept of a graph tour.
- 7) Given a graph, construct a set of test paths through the graph
- 8) Construct a graph from a segment of source code.

Graph Definitions

- A set N of nodes, N is not empty 
Circle
- A set N_0 of initial nodes, N_0 is not empty
Starting nodes A node w/ arrow coming in
- A set N_f of final nodes, N_f is not empty
Ending node w/ no outbound edges
- A set E of edges, each edge from one node to another 
 - (n_i, n_j) , i is predecessor, j is successor

Arrows

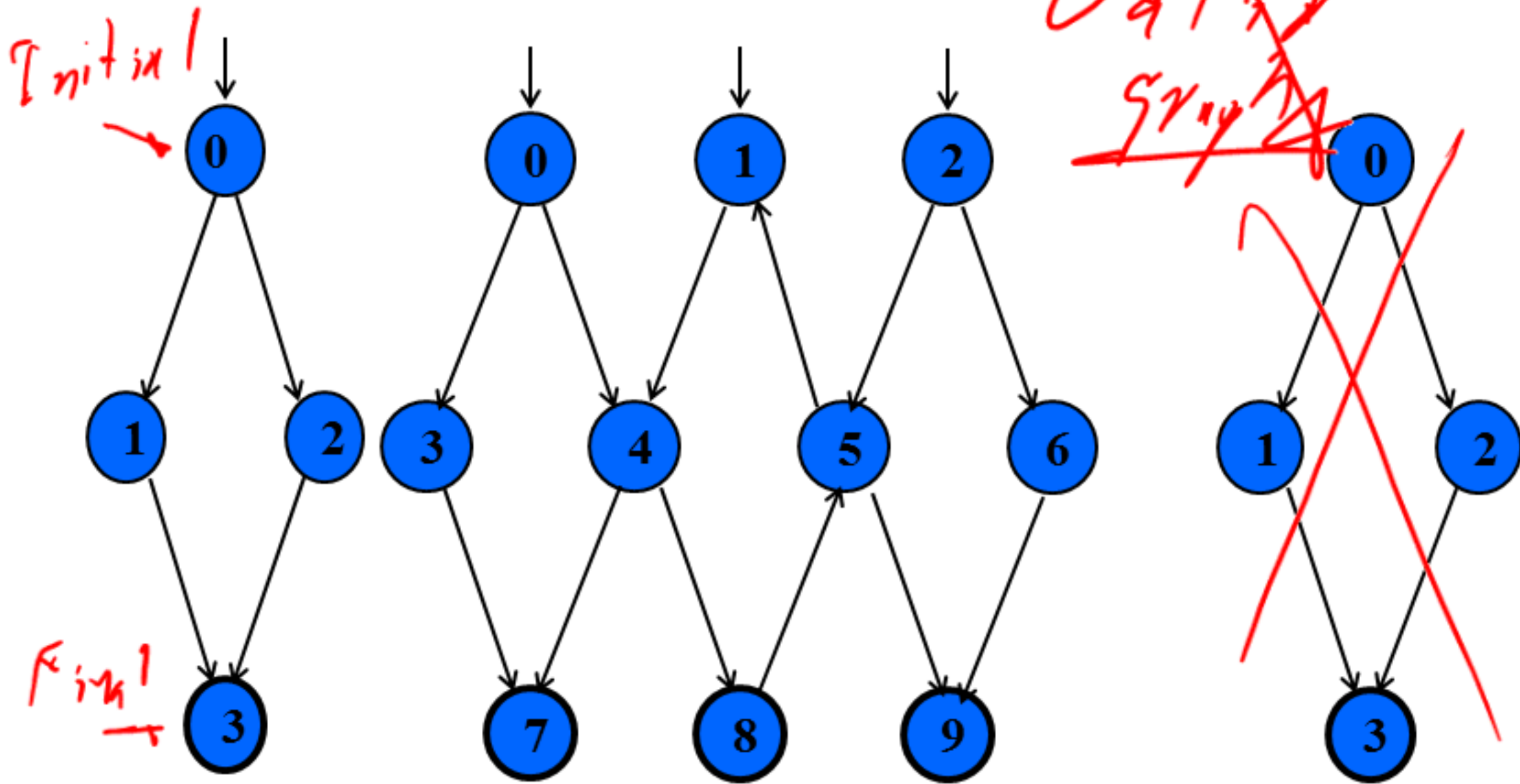
Example Graphs



2 initial Node: A

Final Node: C, D

Three Example Graphs



$$N_0 = \{ \underline{0} \}$$

$$N_f = \{ 3 \}$$

$$N_0 = \{ 0, 1, 2 \}$$

$$N_f = \{ 7, 8, 9 \}$$

$$N_0 = \{ \}$$

$$N_f = \{ 3 \}$$

Multiple initial

Multiple final nodes



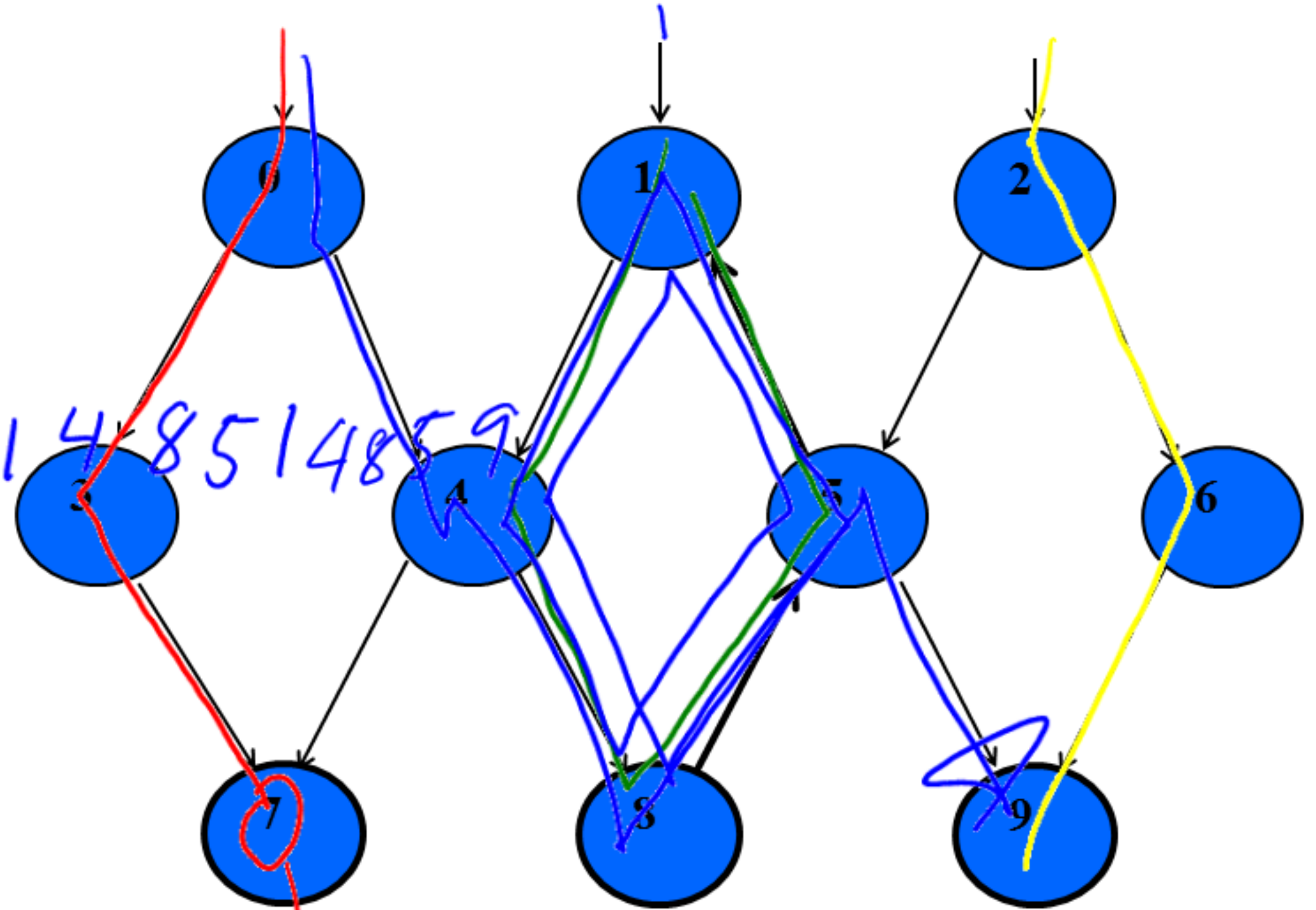
How long the path is
Paths in Graphs

- Path : A sequence of nodes – $[n_1, n_2, \dots, n_M]$
 - Each pair of nodes is an edge *How we walk through*
- Length : The number of edges *the graph.*
 - A single node is a path of length 0
- Subpath : A subsequence of nodes in p is a subpath of p *Path through a part of*
- Reach (n) : Subgraph that can be reached from n *path.*
- Reachable
 - Indicates that there exists a path from node i to node j *⇒ can get there*



Path 5:
0,3,7
1,4,8,5,1
2,6,9

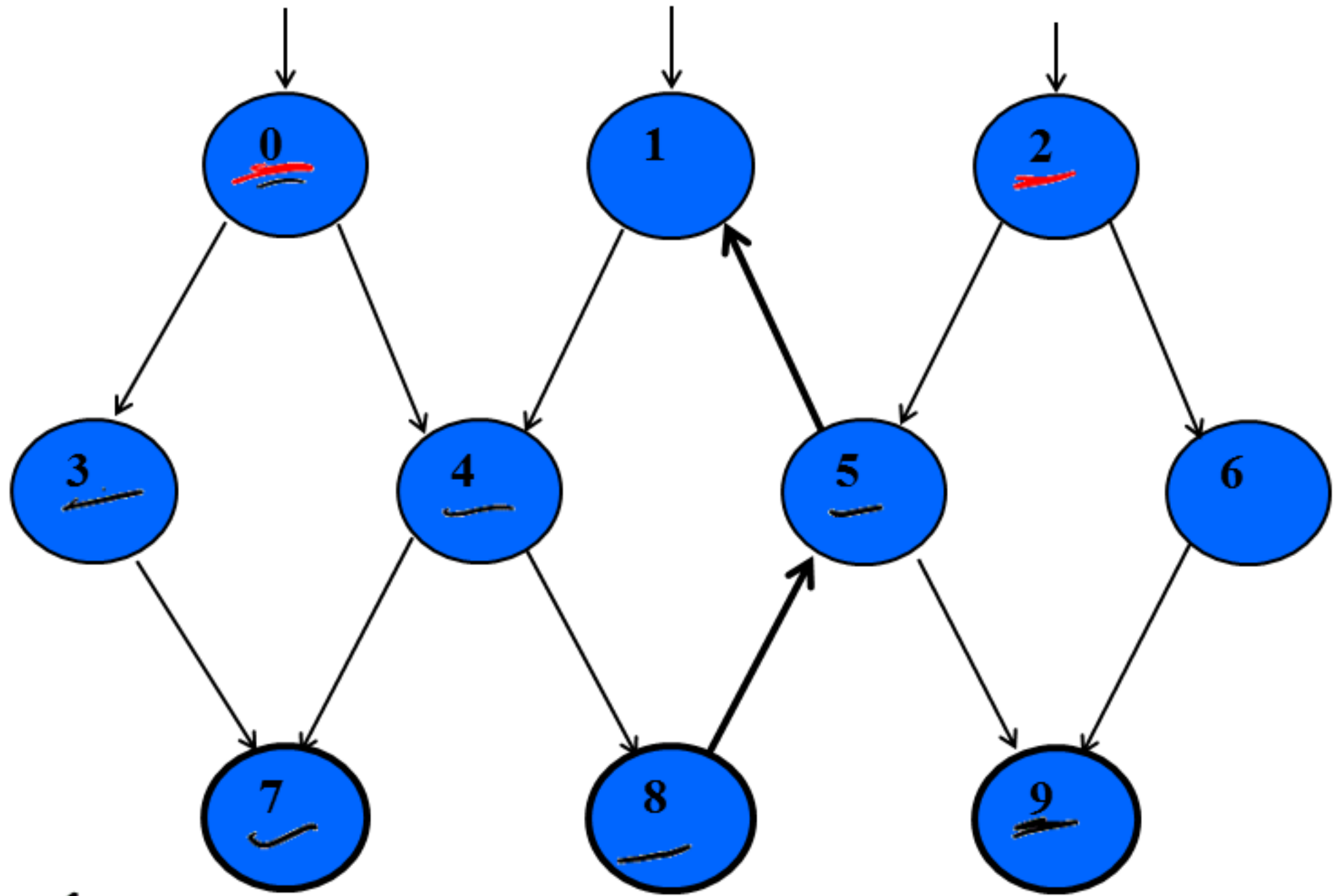
A Graph



Final Node



A Graph



Reach

$$\text{Reach}(0) = \{0, 3, 4, 7, 8, 5, 9, 1\}$$

$$\text{Reach}(\{0, 2\}) = G$$

Test Path

- Test Path : A path that starts at an initial node and ends at a final node
- Test paths represent execution of test cases
 - Some test paths can be executed by many tests
 - Some test paths cannot be executed by any tests

walks through graph

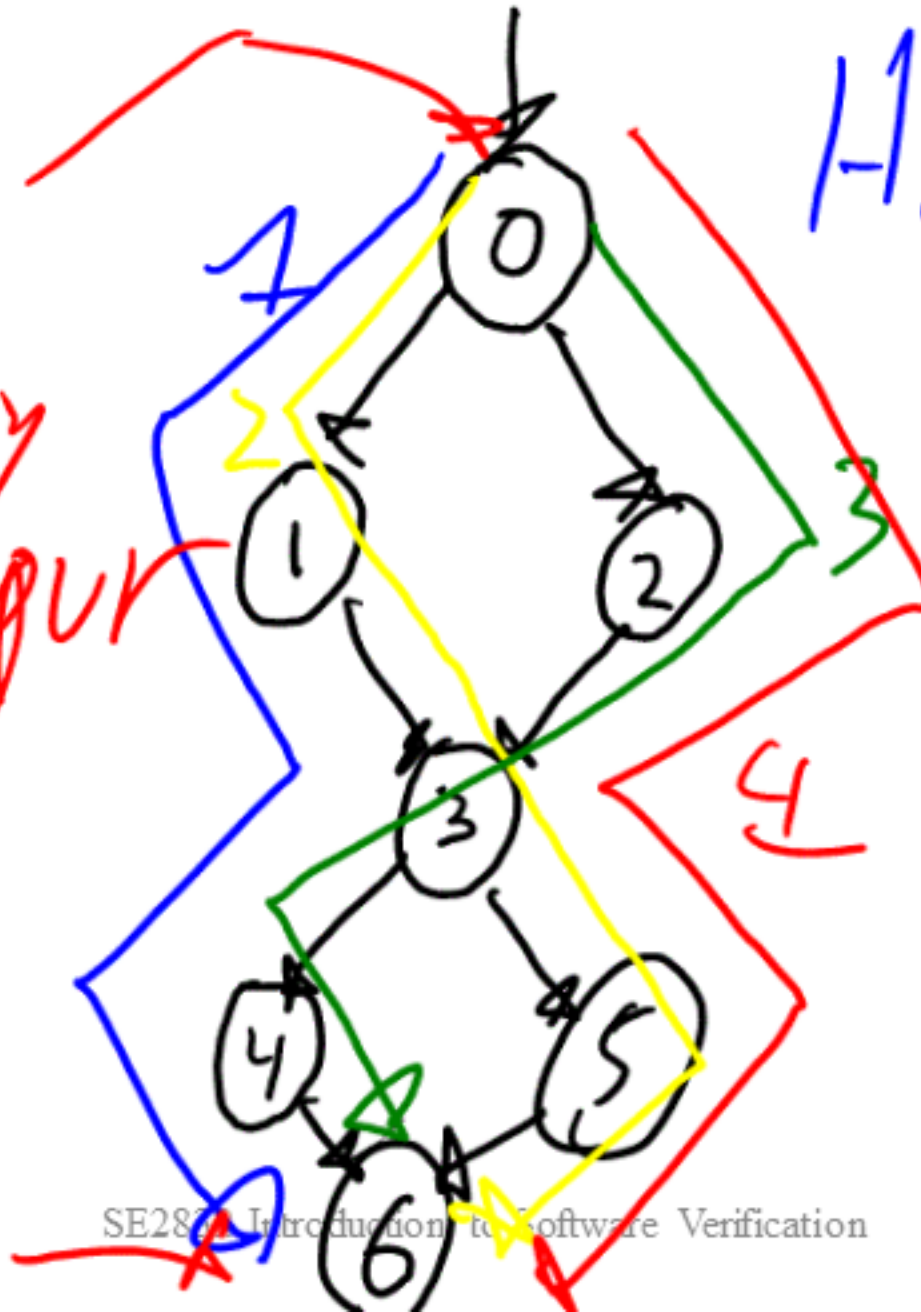
- SESE graphs: All test paths start at a single node and end at another node

- Single-entry, single-exit
- N_0 and N_f have exactly one node
- Ideally what we want to see in software

SESE Graph

Single entry
Entry in the method

return



How many paths?

4

4

Visiting and Touring

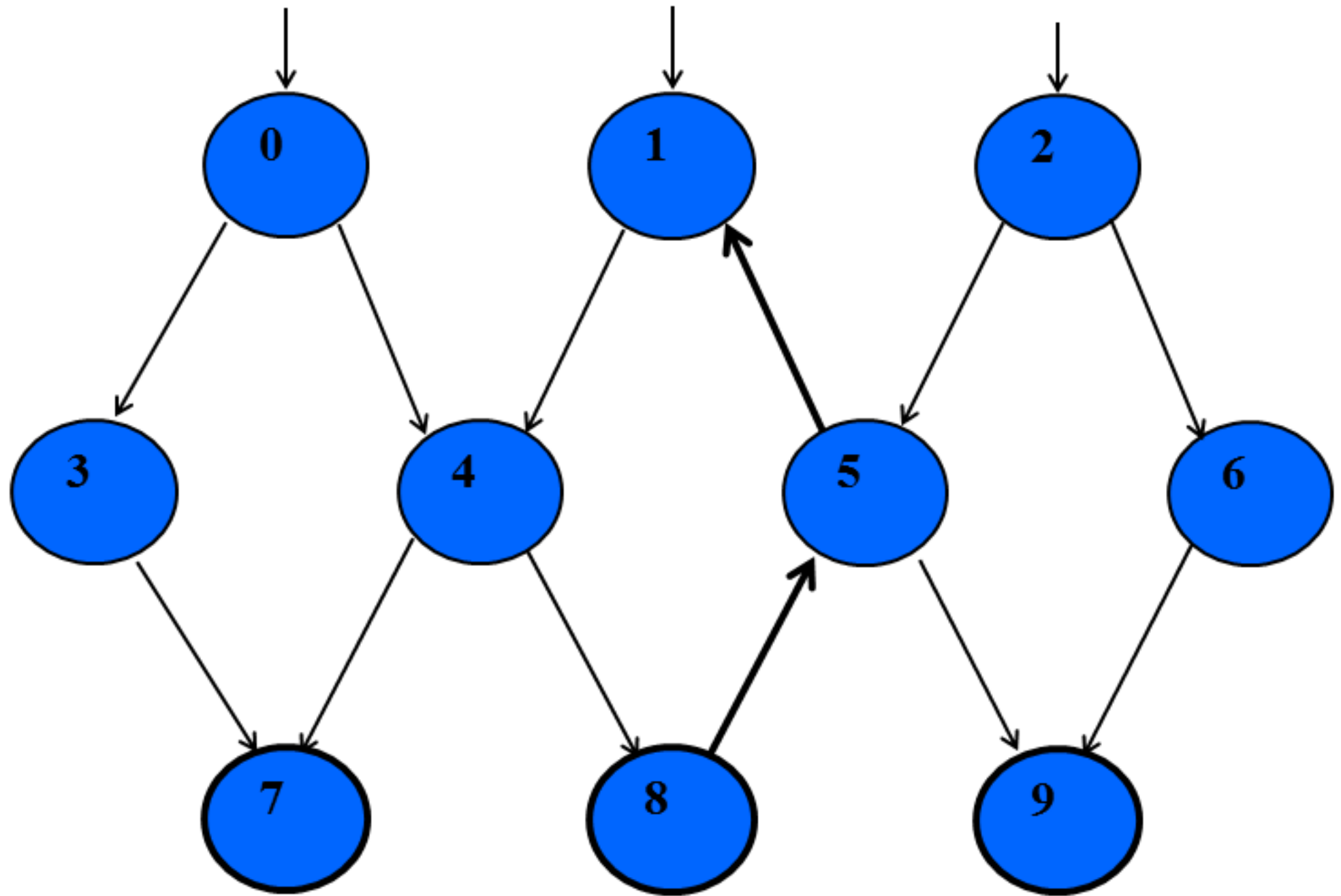
- Visit :
 - A test path p visits node n if n is in p
 - A test path p visits edge e if e is in p
- Tour :
 - A test path p tours subpath q if q is a subpath of p

test path

covers

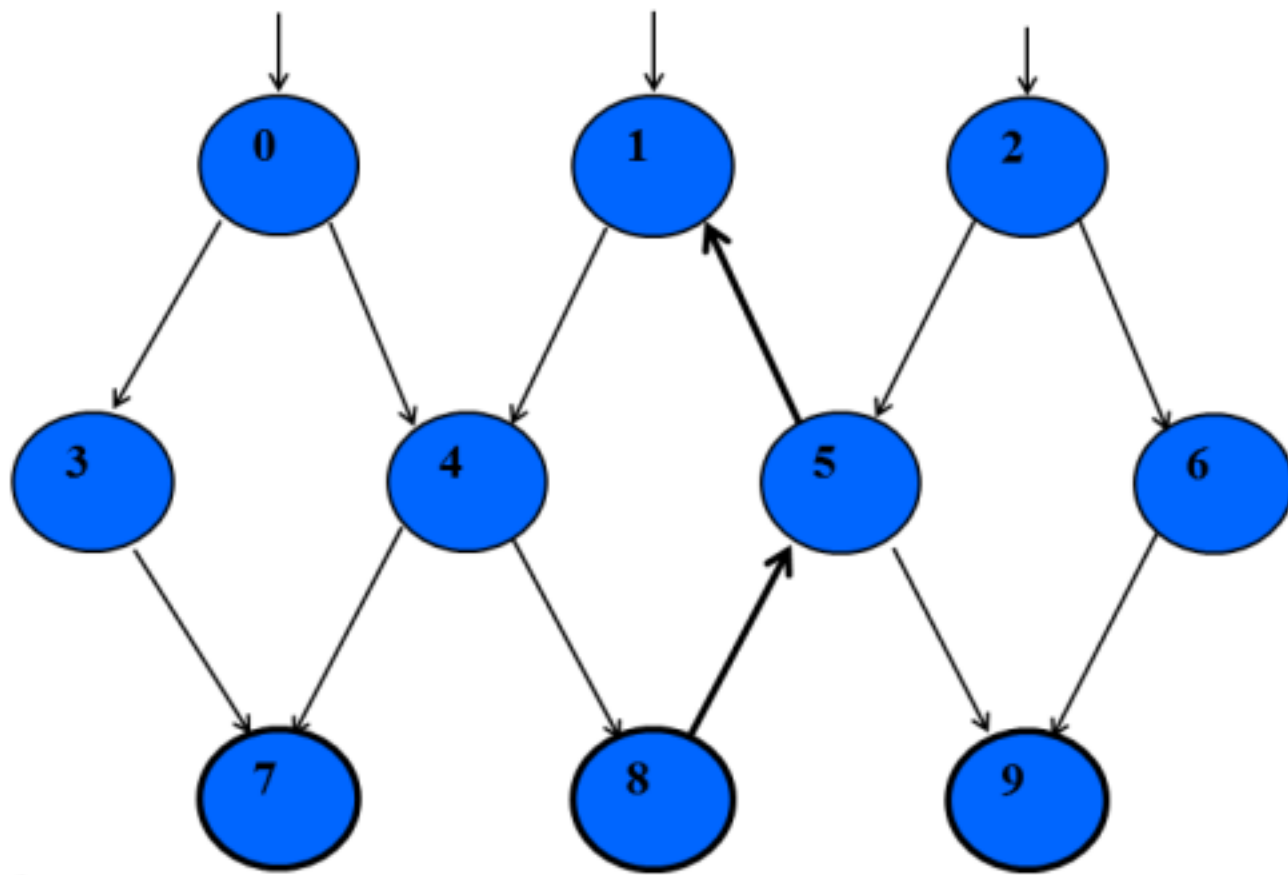


A Graph



Path 0 1 3 4 6
Visited Nodes: 0 1 3 4 6

A Graph



Path: 0 4 8 5 9

Nodes Visited: 0, 4, 8, 5, 9

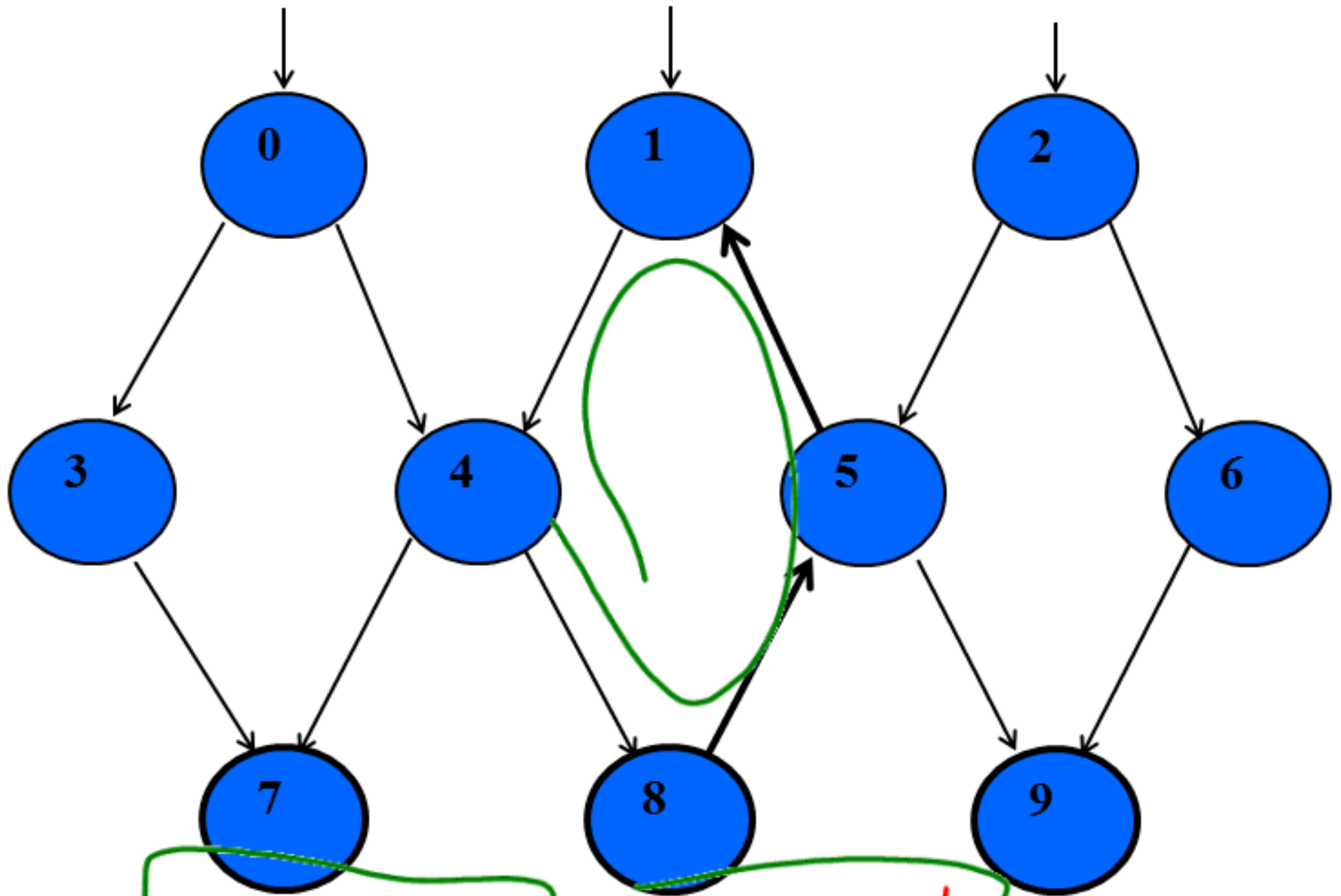
Edges Visited: (0,4) (4,8) (8,5) (5,9)

SE2832 Introduction to Software Verification



Subpaths tour: 0 4 8 4 8 5
0 4 8 5 8 5 9

A Graph



Path: 0

4 8 5 1

4 8 5 1

4 8 5 9

Subpath

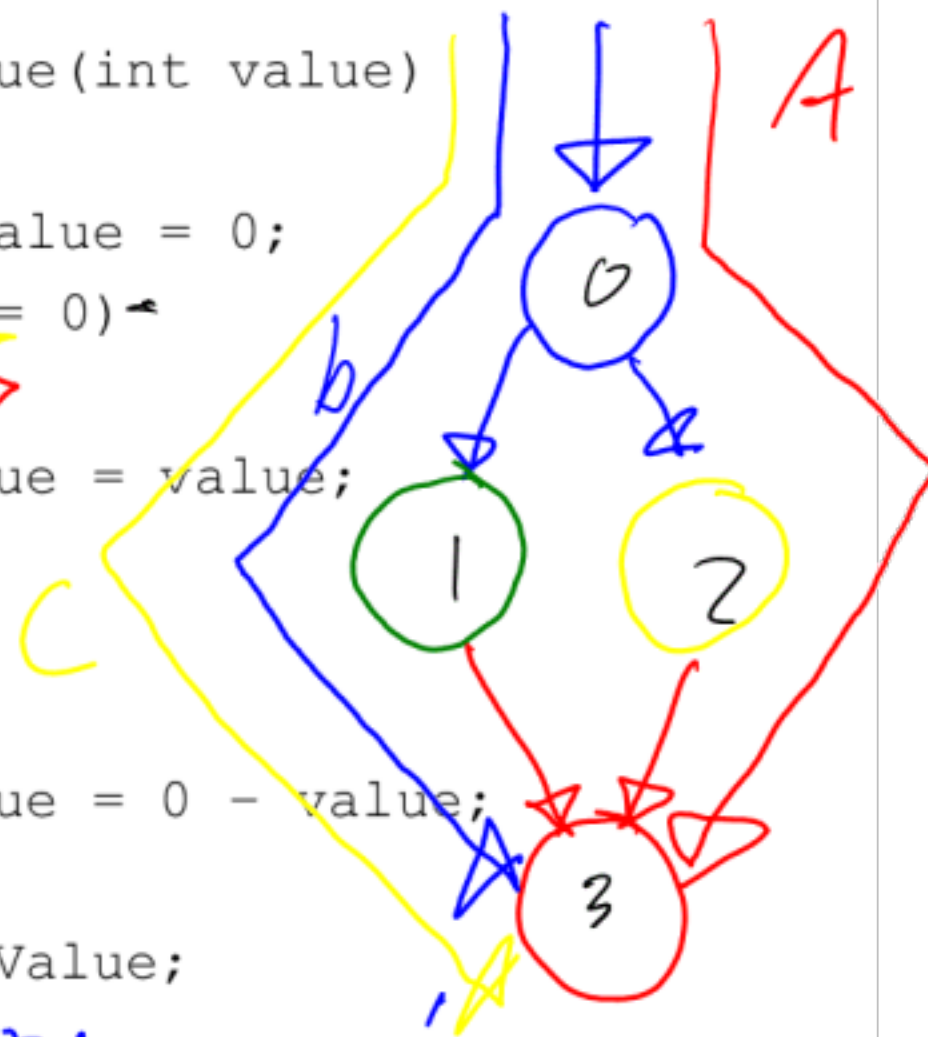
8 5 1 4

Converting source code to a

```
int absoluteValue(int value)
{
  int returnValue = 0;
  if (value >= 0)
  {
    returnValue = value;
  }
  else
  {
    returnValue = 0 - value;
  }
  return returnValue;
}
```

~~Way to graph~~

$(0,0) \rightarrow$



Control Flow graph

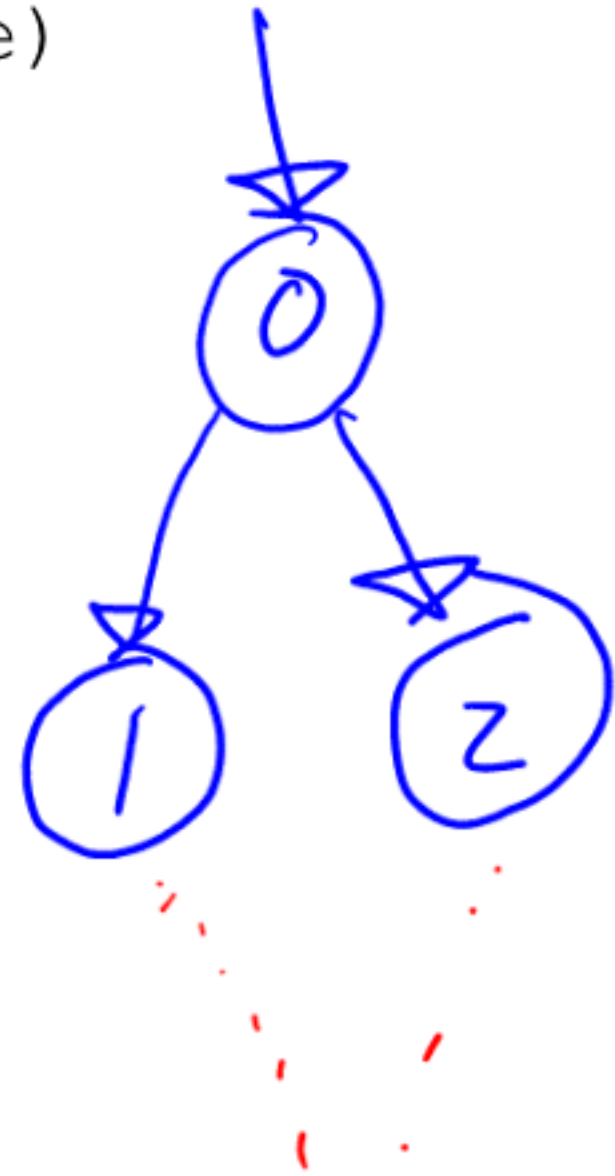


Test cases: $(-5, 5)$ - A
 $(5, 5)$ - B
 $(0, 0)$ - C

Converting source code to a

graph

```
boolean isOdd(int value)
{
    if (value%2 != 0)
    {
        return true;
    }
    else
    {
        return false;
    }
}
```



Not SE SE calling print

```
int Mod 5 (int val)
{
  if (val % 5 == 0)
  { return 0; }
  else if (val % 5 == 1)
  { return 1; }
  else if (val % 5 == 2)
  { return 2; }
  else if (val % 5 == 3)
  { return 3; }
  else return 4;
}
```

