



Graph Theory

Lecture Objectives:

- 1) Define the term graph, node, initial node, final node, and edge.
- 2) Define reachability in a graph.
- 3) Define the term test path.
- 4) Explain the concept of a SESE graph.
- 5) Relate the concept of a SESE graph to good programming concepts.
- 6) Define the concept of visiting a node and explain the concept of a graph tour.
- 7) Given a graph, construct a set of test paths through the graph.
- 8) Construct a graph from a segment of source code.

Graph .

Lines and

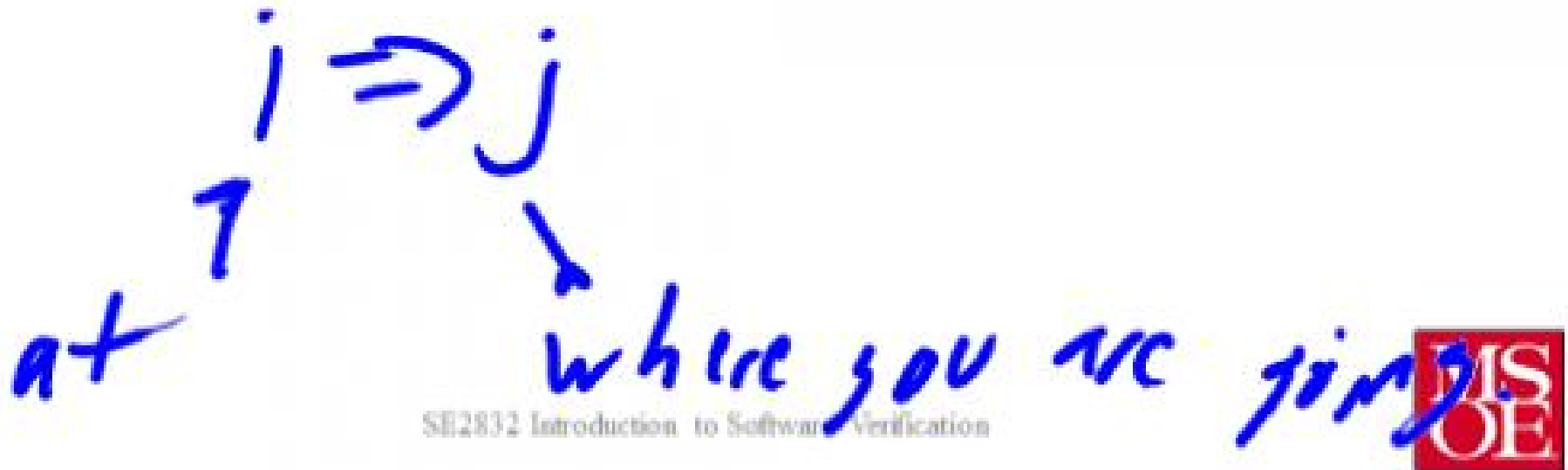
Nodes, Edges



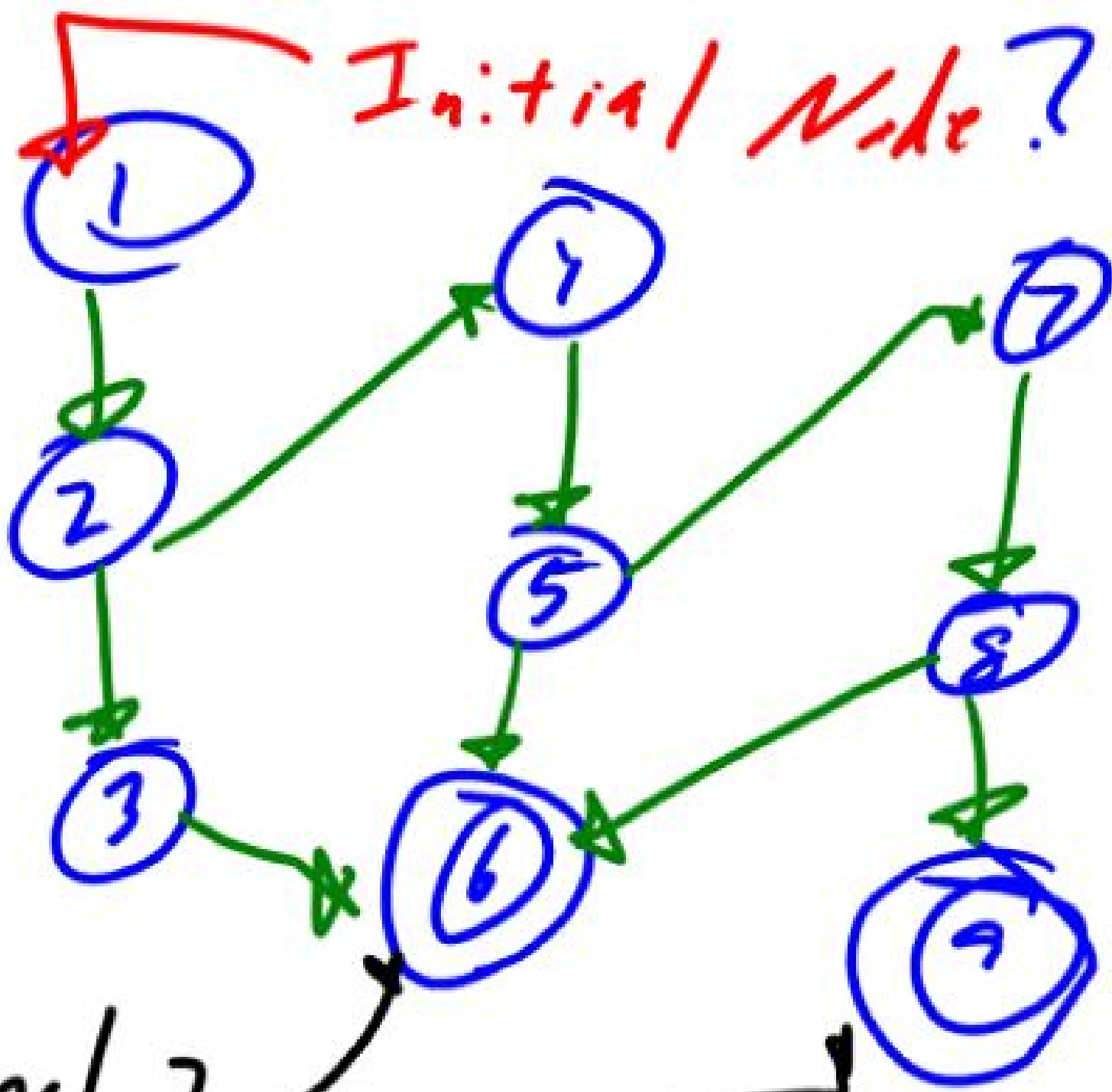
Vertices ...

Graph Definitions


- A set N of nodes, N is not empty
- A set N_0 of initial nodes, N_0 is not empty *Starting points*
- A set N_f of final nodes, N_f is not empty *Finishing points*
- A set E of edges, each edge from one node to another
 - (n_i, n_j) , i is predecessor, j is successor

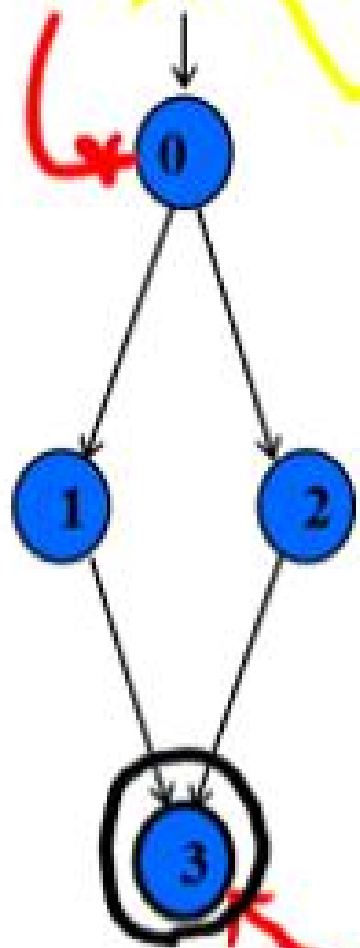


Example Graphs

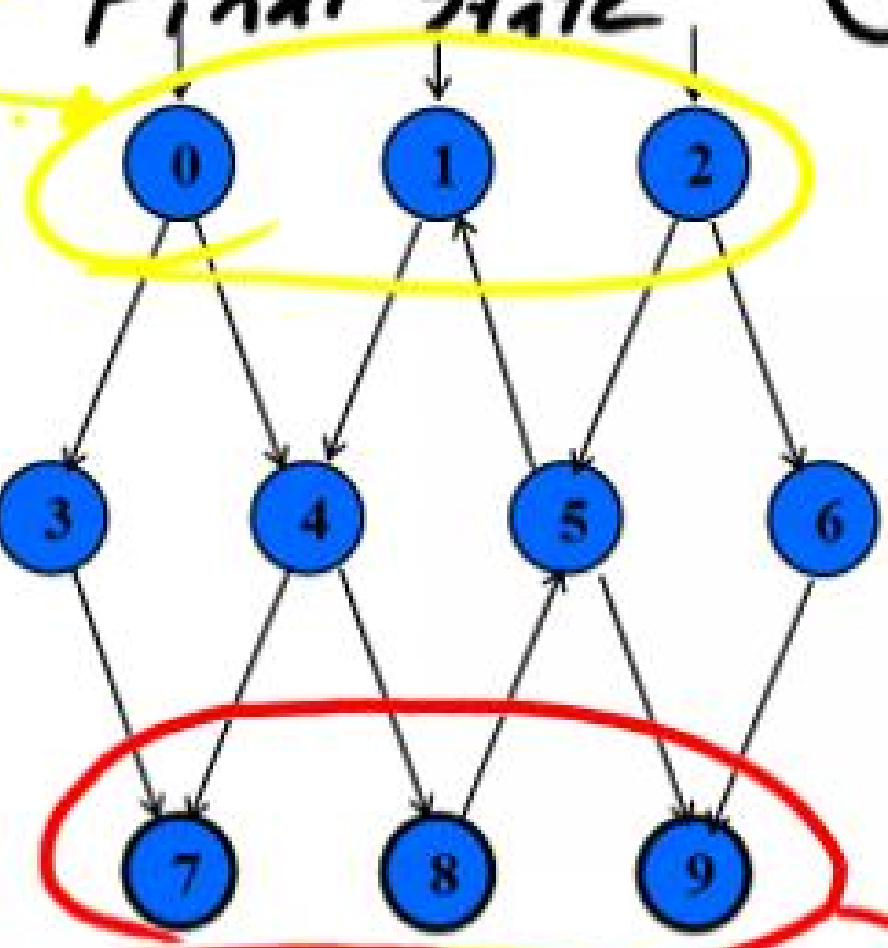


Three Example Graphs

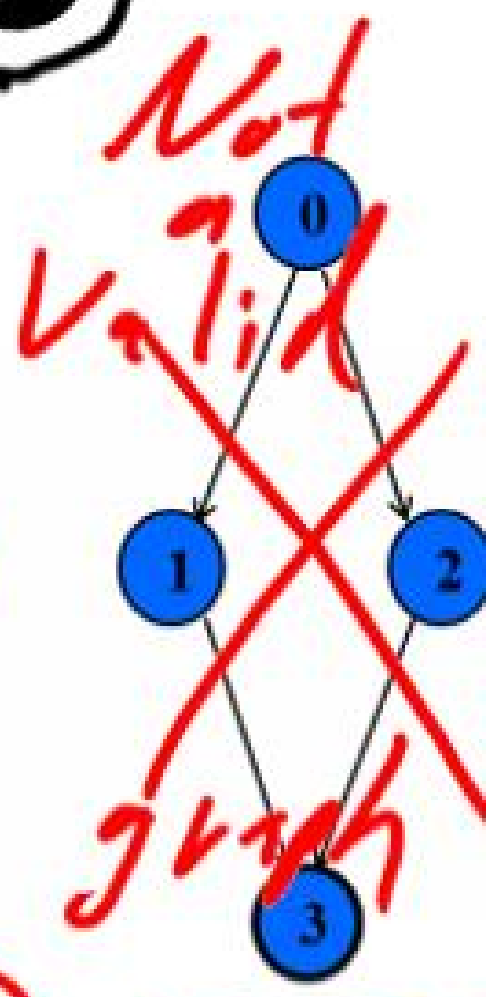
Initial STATES \rightarrow VML \rightarrow Final State \rightarrow 



$N_0 = \{0\}$
 $N_f = \{3\}$



$N_0 = \{0, 1, 2\}$
 $N_f = \{7, 8, 9\}$



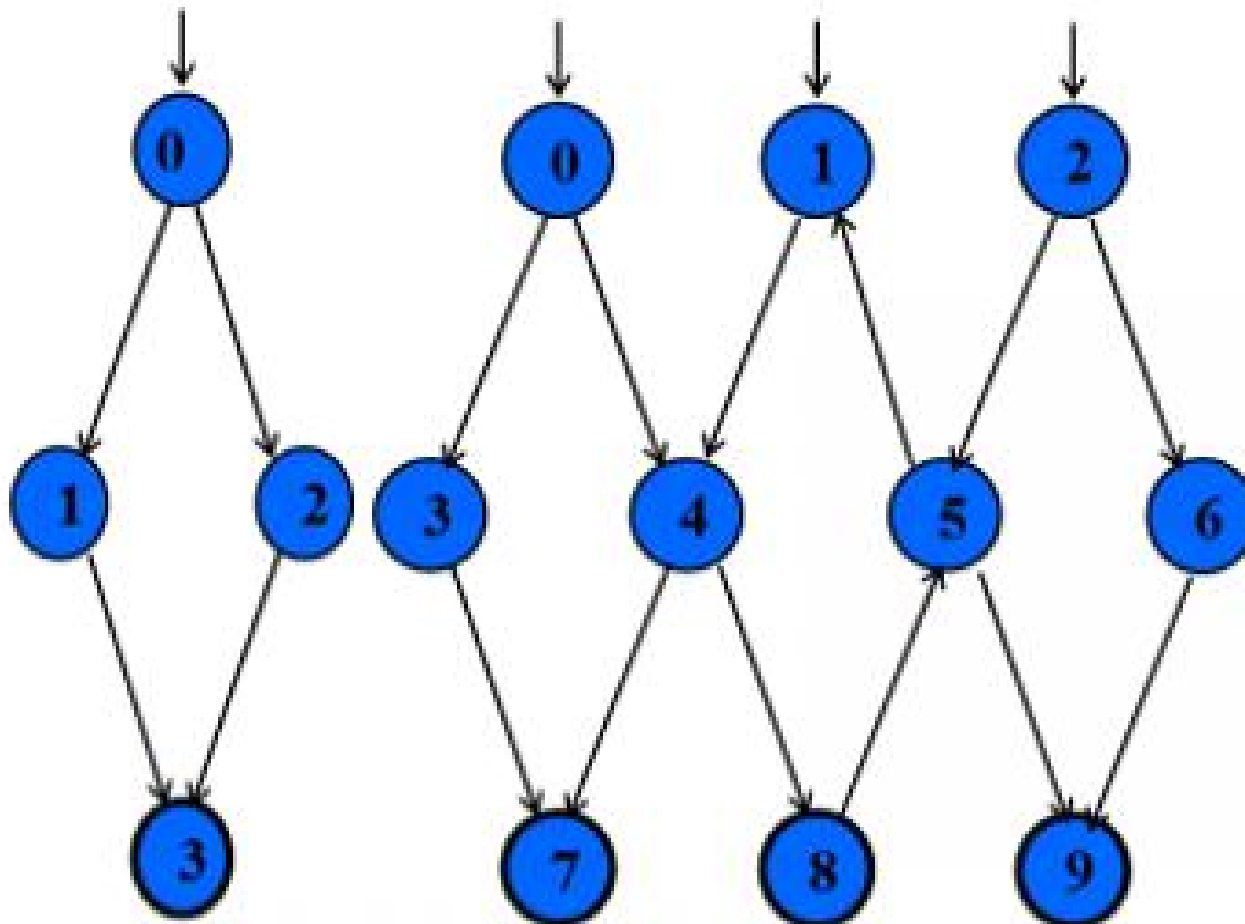
Not Valid graph

$N_0 = \{\}$
 $N_f = \{3\}$

Not No initial



Three Example Graphs



$$N_0 = \{0\}$$

$$N_f = \{3\}$$

$$N_0 = \{0, 1, 2\}$$

$$N_f = \{7, 8, 9\}$$

$$N_0 = \{0\}$$

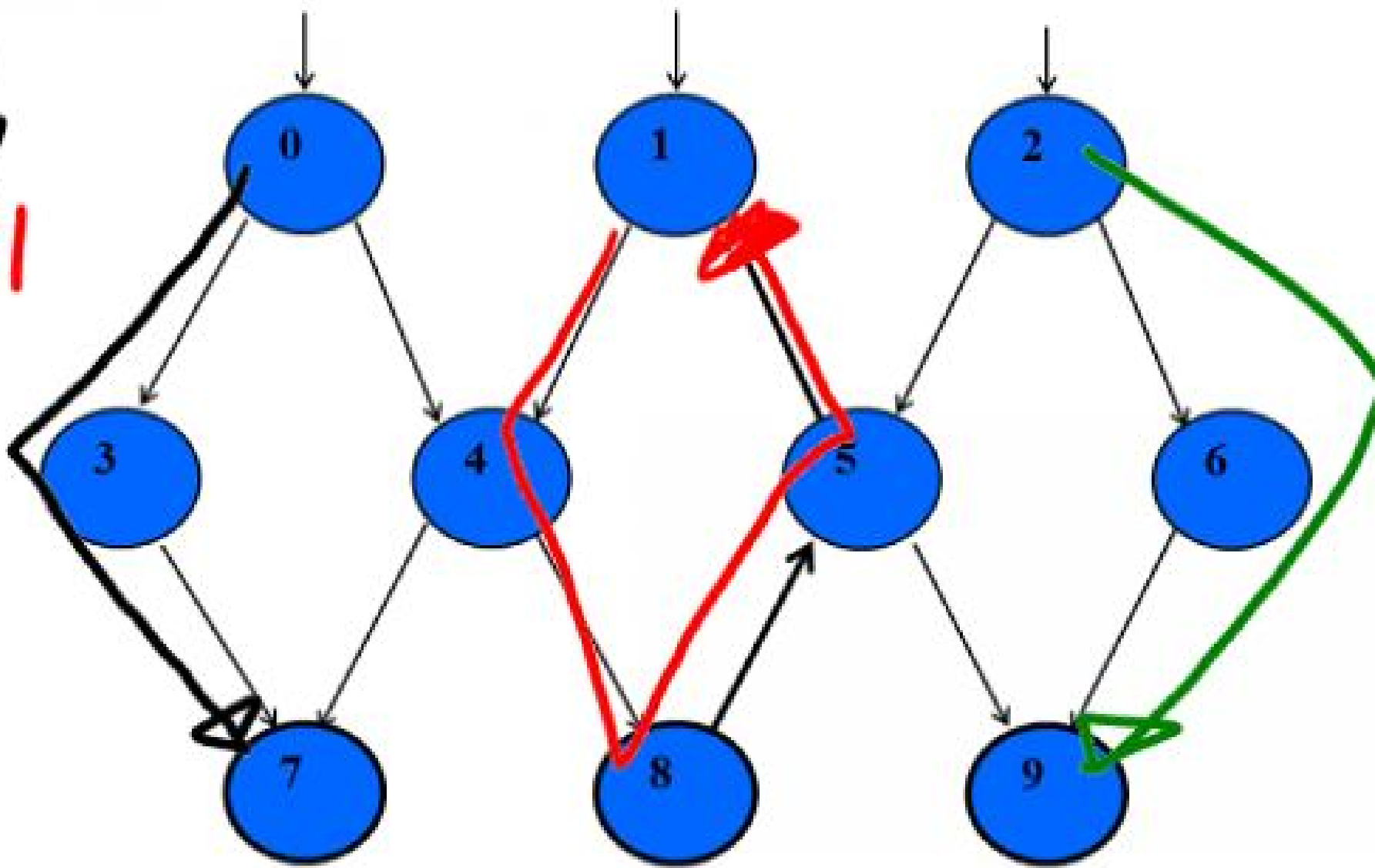
$$N_f = \{3\}$$

Paths in Graphs

- Path : A sequence of nodes – $[n_1, n_2, \dots, n_M]$
 - Each pair of nodes is an edge
- Length : The number of edges \Rightarrow How long is a path?
 - A single node is a path of length 0
- Subpath : A subsequence of nodes in p is a subpath of $p \Rightarrow$ Part of a path
- Reach (n) : Subgraph that can be reached from n
- *Reachable*
 - Indicates that there exists a path from node i to node j .

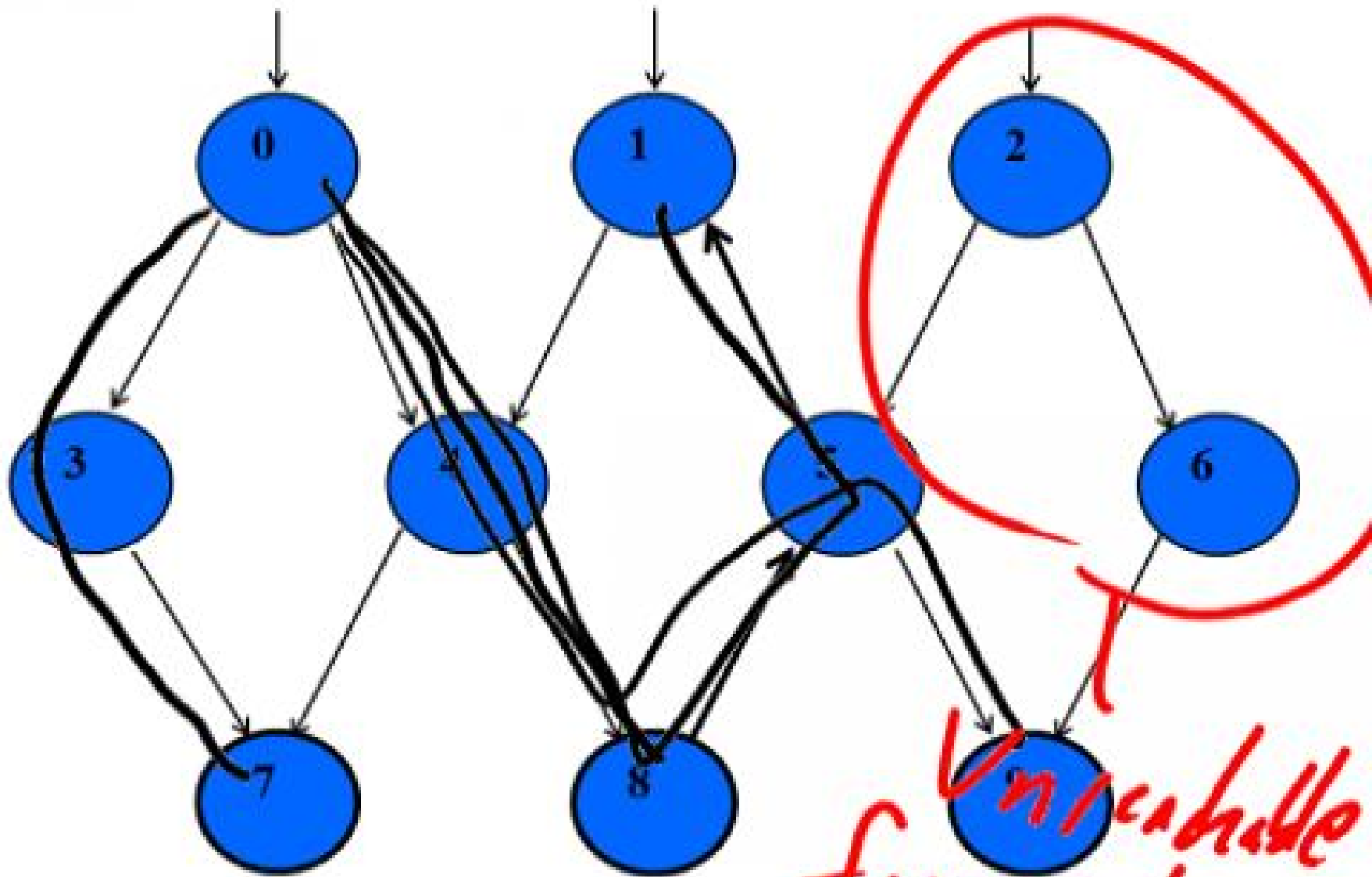
Path
0 3 7
1 4 8 5 1
2 6 9

A Graph



Reach

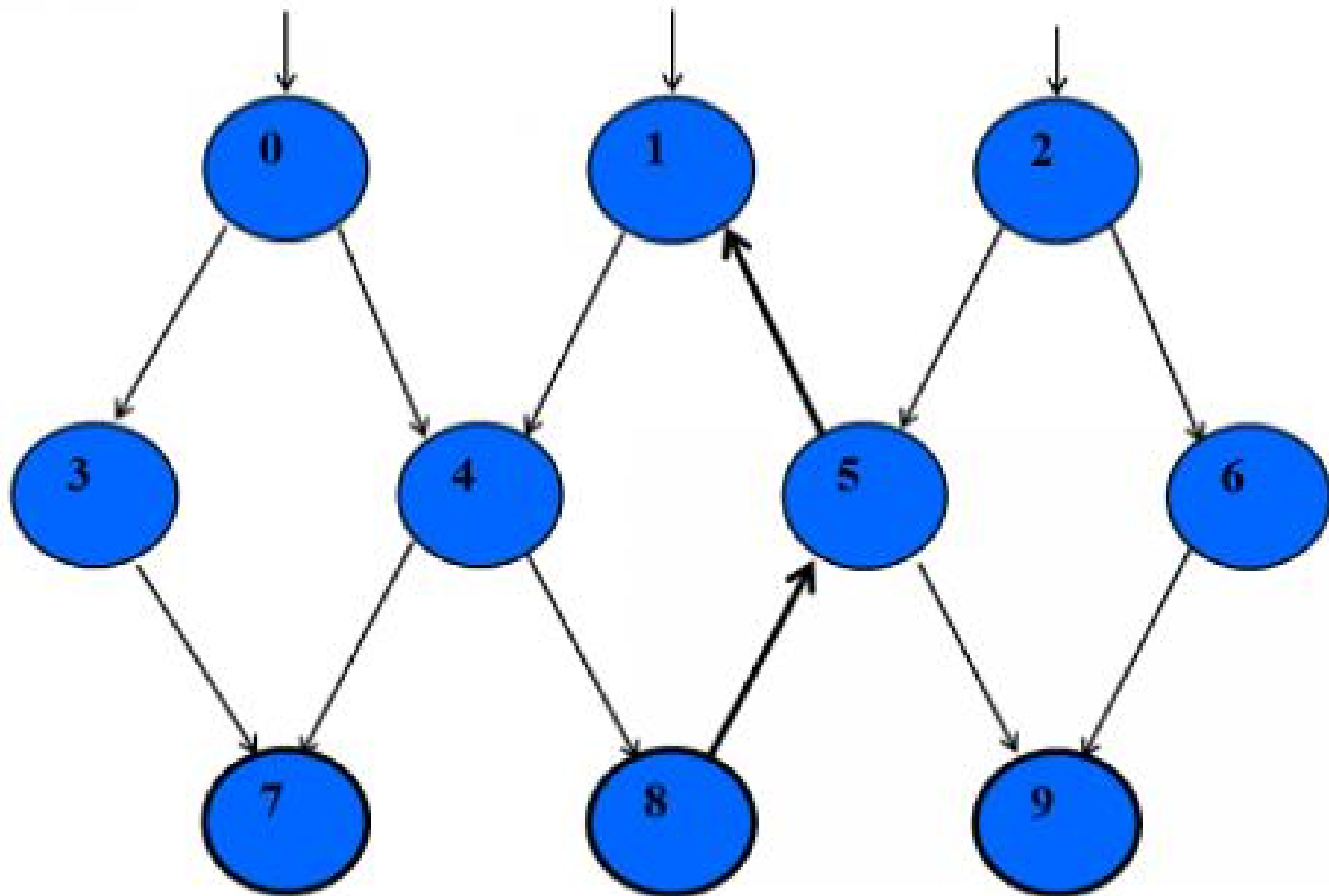
A Graph



Node 0 \Rightarrow 3 7 4 8 5 1 7.

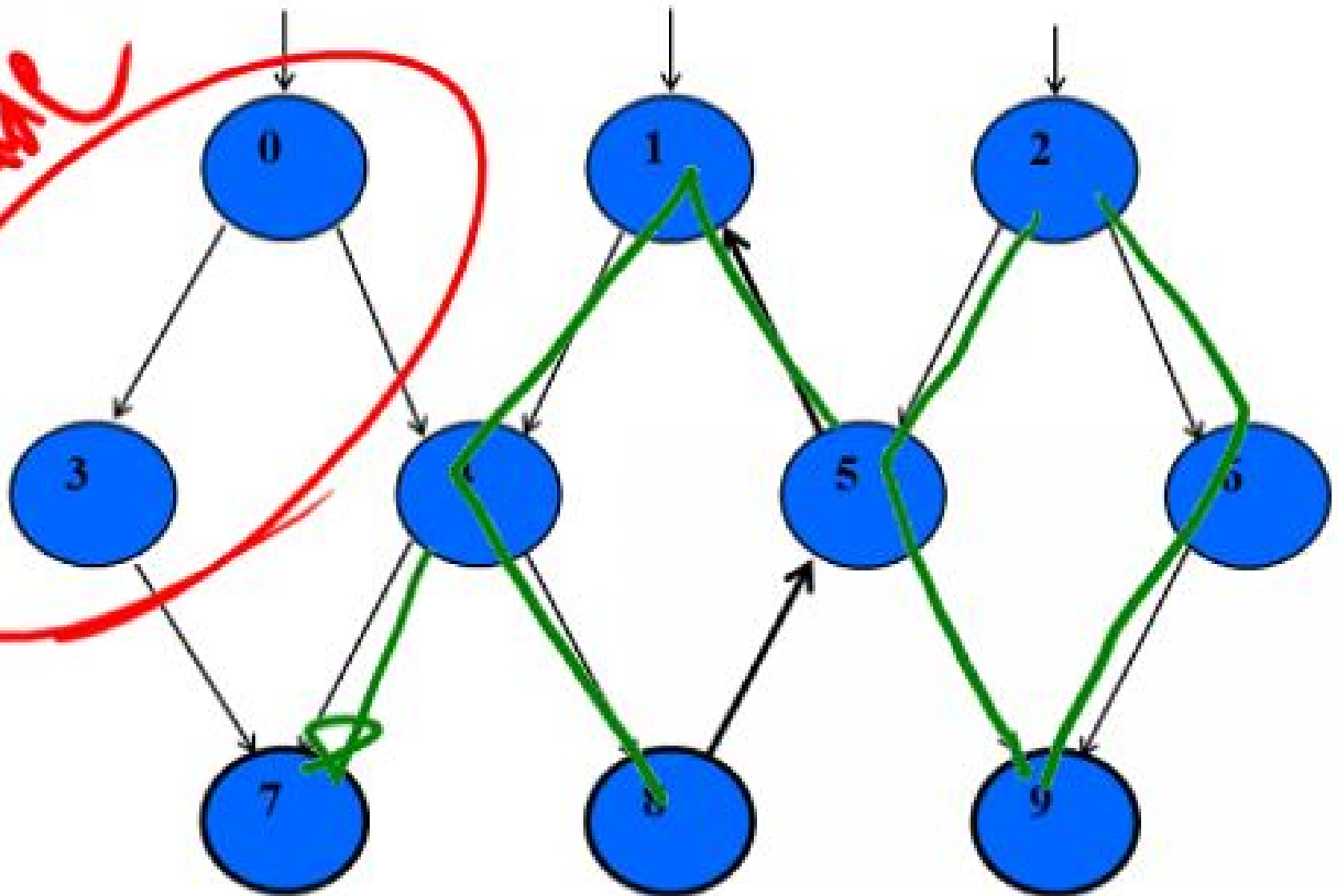
Unreachable
from node 0.

A Graph



$\text{Reach}(0, 2) \Rightarrow$ All nodes
(G)

A Graph \checkmark Reachable



Reach(2,6) \Rightarrow 2, 6, 9
148 \rightarrow

Test Path

- Test Path : A path that starts at an initial node and ends at a final node
- Test paths represent execution of test cases
 - Some test paths can be executed by many tests ✓
 - Some test paths cannot be executed by any tests ✗

Tests to be
Bad

- SESE graphs : All test paths start at a single node and end at another node

4 test paths

- Single-entry, single-exit
- Nf and Nt have exactly one node
- Ideally what we want to see in software

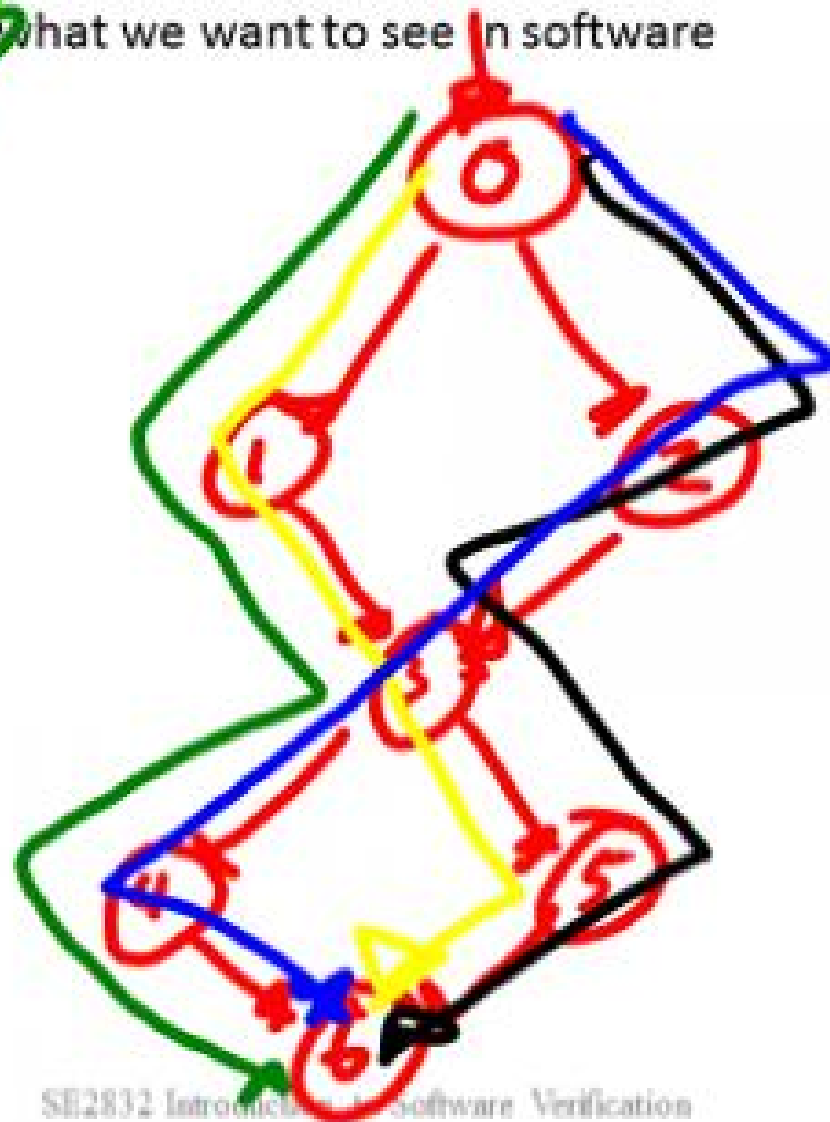
SESE Graph

0 1 3 4 6

0 3 5 6

0 3 5 6

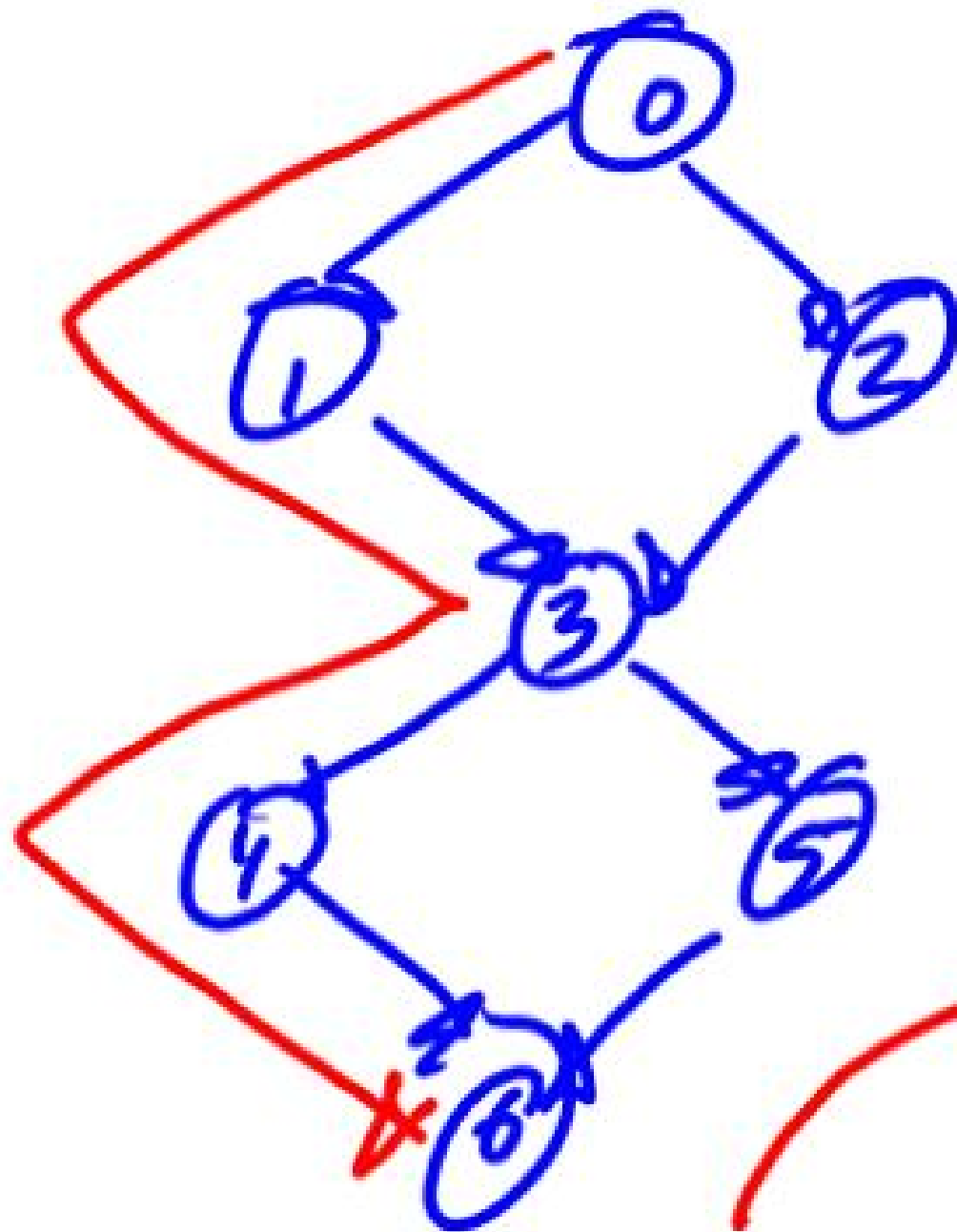
0 2 3 4 6



Visiting and Touring

- Visit :
 - A test path p visits node n if n is in p
 - A test path p visits edge e if e is in p
- Tour :
 - A test path p tours subpath q if q is a subpath of p



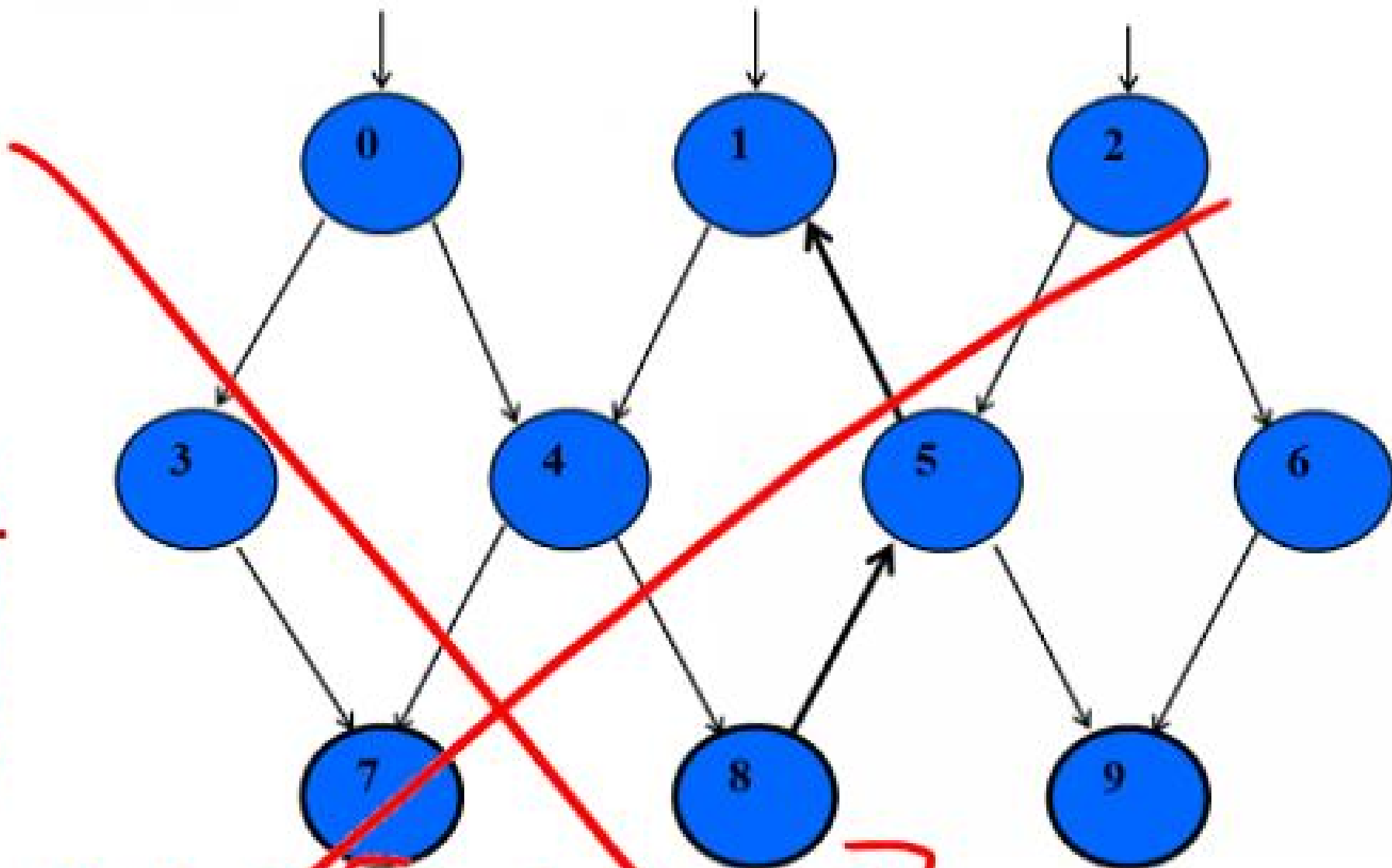


$[3, 4, 6]$

Tours

subpath $[0, 1, 3]$

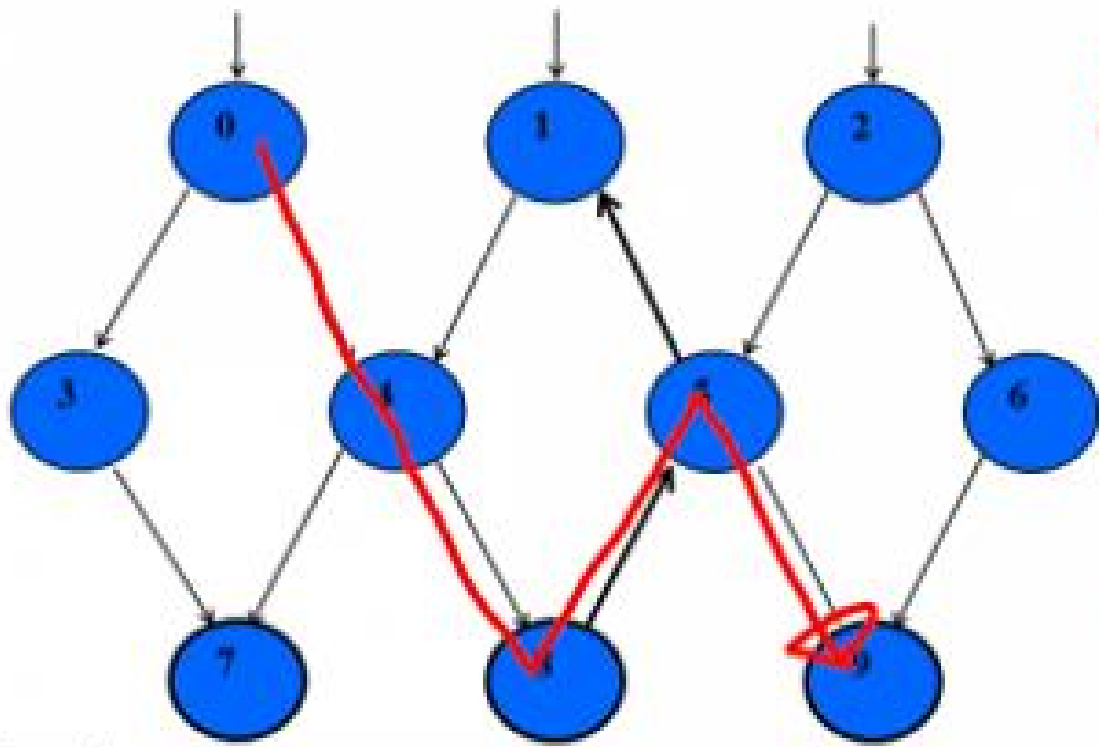
A Graph



Path: [0 1 3 4 6]

What nodes are visited?

A Graph



Which
Subpaths
were toured?

Path: 0 4 8 5 9
Nodes Visited: 0 4 8 5 9

(0 4 8) (5 8 9)
(4 8 5)
(8 5 9)
(0 4 8 5)

Visited which edges?

(0 4) (4 8) (8 5) (5 9)



Converting source code to a

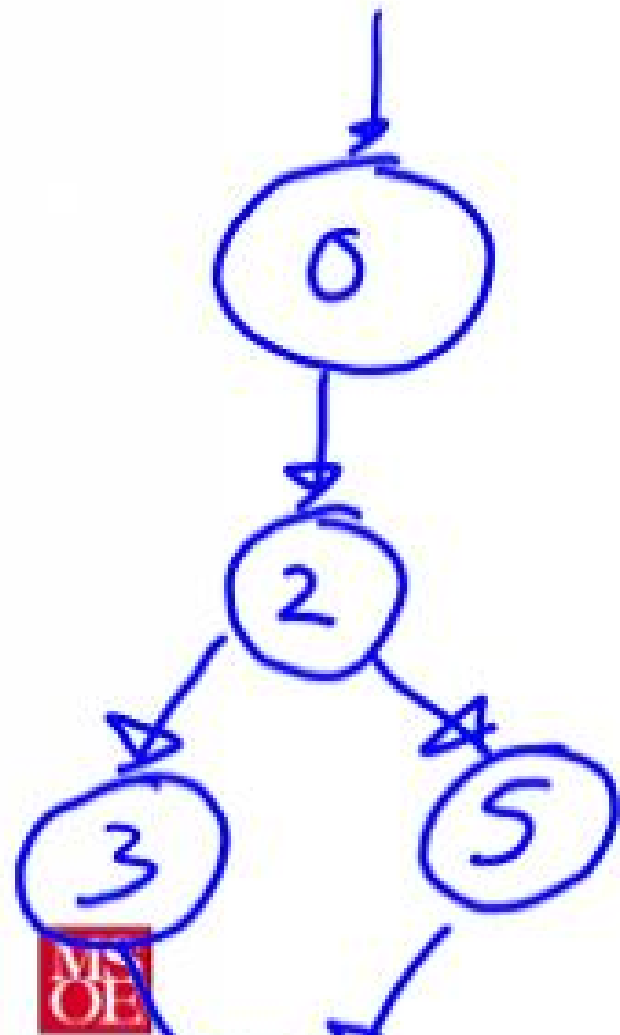
graph

```
int absolute*value(int value)
{
    int returnValue = 0;
    if (value >= 0)
    {
        returnValue = value;
    }
    else
    {
        returnValue = 0 - value;
    }
    return returnValue;
}
```

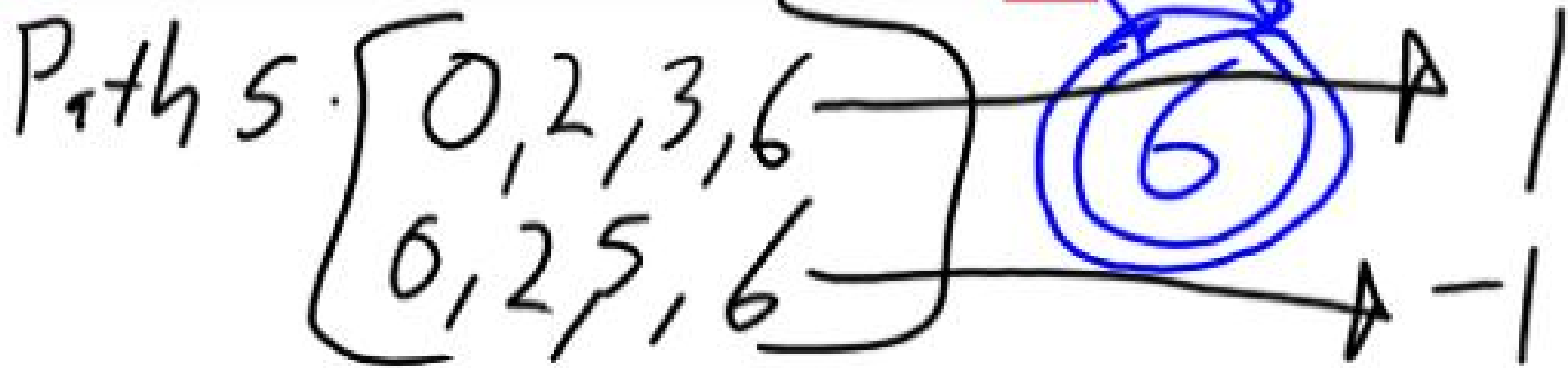
Converting source code to a graph

graph

```
0 int absoluteValue(int value)
  {
  1 int returnValue = 0;
  2 if (value >= 0)
  {
  3   returnValue = value;
  }
  4 else
  {
  5   returnValue = 0 - value;
  }
  6 return returnValue;
  }
```



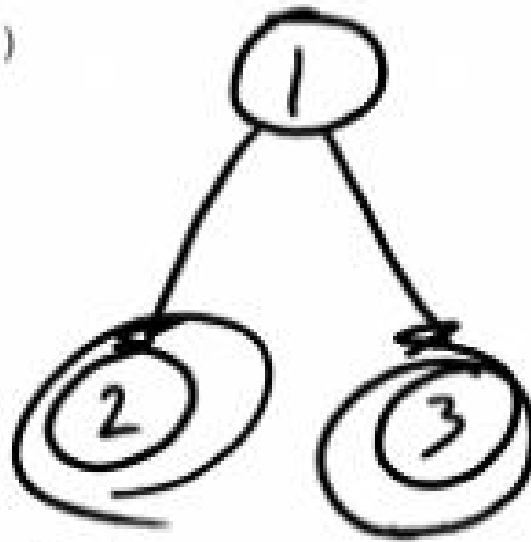
©2012 Introduction to Software Verification



Converting source code to a graph

graph

```
boolean isOdd(int value)
{
  1 if (value%2 != 0)
  {
    2 return true;
  }
  else
  {
    3 return false;
  }
}
```



$$(1+2) \Rightarrow 3$$
$$(1+3) \Rightarrow 2$$



NOT
SENSE

